

On Redistributive Effects of Monetary Shocks

Considerations for Developing Countries

Emmanuel Marino Casuscelli

QMSS G5999 Masters Thesis

Columbia University

May 9th, 2008

Quantitative Methods in the Social Sciences Program
Institute for Social & Economic Research & Policy
Columbia University in the City of New York
International Affairs Building
420 West 118th Street
8th Floor, mail code 3355
New York, New York 10027

0. Introduction

This paper proposes two redistributive mechanisms that arise out of the convergence between an exogenous monetary shock and asymmetries typically endogenous to developing and less developed nations. The sudden increases in a nation's money supply can have highly destabilizing short-run consequences for the economy. During such periods of uncertainty and readjustment, those that react quickly enough can take advantage of opportunities to profit from the circumstances. And it is exactly the ability to react in such circumstances that is dependent on which side of the asymmetric coin an individual finds himself on.

The first and fundamental asymmetry that we focus on is purely informational in nature. It is nurtured by the stark differences in even basic education that are ubiquitous to many developing countries. With this in mind, we assume that individuals who are more educated can better analyze an economic environment than those who lack elementary analytical skills imparted by formal education. We operationalize this difference via differing expectations on the future behavior of the economy in the face of some type of economic shock. One group, the educated group, we will assume holds perfect foresight, and well understands the functioning of the economy according to a basic set of rules. The second, uneducated group, we will assume has lagging expectations, and only reacts to macroeconomic shocks well *after* the educated group has acted. We note that both groups are rational; however, one group has a better framework with which to analyze existing economic information. This leads to arbitrage opportunities in favor of the educated group, who can prey on the naïveté of less informed participants insofar as the 'correct' pricing of assets is concerned.

The second asymmetry we draw attention to is born out of differences in access to capital markets (i.e. imperfect capital markets). In our second model, we keep the asymmetries present in the first model and allow the group of educated agents to invest in interest bearing assets, while the less privileged group can only access cash. This situation is common in many developing, where a privileged few can invest in a small, and limited financial system, while the rest of the often rural economy is left to holding cash balances only. Given a one-time permanent increase to the money supply, and a decrease in the interest rate, the privileged group will see the nominal value of their financial instrument (e.g. a bond) go up in the short run. Thus, this group gets an *additional* nominal appreciation of his wealth. If we combine this wealth effect with the aforementioned differences in

expectations, and the privileged group has *even more* wealth to invest into the purchase of the naively undervalued real assets of the uninformed agents.

The underlying problem we evaluate here is inflation, and how individuals and assets react to it. We take the money supply to be exogenous, and inflation to be consequently endogenous. An unanticipated money supply shock is the ultimate causal factor in this model. However, given a monetary shock, some agents –given aforementioned informational advantages– can anticipate inflation. This, coupled with, the distinction between real and nominal assets that leads to an eventual redistribution of wealth. Since real assets rise in price as prices themselves rise, they are an attractive hedge against inflation. If the nominal quantity of money increases, nominal assets, like cash, lose real value relative to real assets as prices begin to rise. As a result, before prices begin to rise, informed agents will rebalance their wealth portfolios to include more real rather than nominal assets.

We make it a point to distinguish between the redistribution of wealth and redistribution of income. In this paper we ignore changes in wealth, i.e. income, and focus on the reallocation of the existing wealth stock in response to a nominal distortion.

1. Literature Review

The topic of wealth distribution in economics is as old as the philosophical and analytical arguments posed by great minds such as Smith, Mill or Marx. The First Theorem of Welfare economics later showed that the allocation of resources does not have to be fair to be Pareto efficient. However, the actual theoretical and empirical study of income distribution at the macro level is relatively recent and can be traced back to Kuznet's (1955) migration-model on the quadratic relationship between income distribution and economic development (Bahmani-Oskooe et al, 2006).

A greater social consciousness and an increased democratization of the global order have brought some of the world's blatant inequalities into the eye of the public community. The subsequent slow accumulation of studies have pointed to causal factors spanning differences in education, technology, the taxation system, land ownership, among a number of others. We choose to focus on the link between changes in the money supply and wealth distribution, and how it interacts with existing social inequalities to further skew the distributive landscape. Focusing thus on the effects of a money shock, i.e. inflation, we can start to narrow our focus on the literature that is most relevant to us.

Most literature on inflation centers on the erosion of the purchasing power of cash as it relates to wages and income. This ‘inflation tax’ often imparted by governments that wish to print money to cancel existing nominal debt obligations (‘seigniorage’) is, if anything, the exogenous factor to which we could attribute the increase in the money supply. Such a motive, as Persson, Persson, and Svensson (1998) note, would initially transfer wealth from private citizens to the government, much like a tax. However, we wish to focus on the direct redistribution of wealth between different sectors of society.

Erosa and Ventura (2002) highlight that the percentage of cash holdings as a portion of total wealth is inversely proportional to income. In other words, the poor hold more cash, while the rich hold more of their wealth in interest bearing financial assets, and real assets. This is interesting to us because it is the end result of our model.

In a very interesting paper Albanesi (2002) poses the question of how the inflation tax is initially set. In her analysis, both the inflation tax and the income tax are set in a political bargaining game between the rich and the poor. Since rich households are less vulnerable to inflation due to the mentioned portfolio compositions, they prefer higher inflation and a lower income tax. Given that the poor are in fact poor, they have a lower bargaining position relative to rich households, who successfully bargain for high money growth rates. Most interestingly for this paper, a cross-country data analysis of the model shows a positive correlation between inequality and average inflation.

Doepke and Schnieder (2003) present a model where they study the role of inflation on the value of nominal assets, and thus look at its real consequences. After collecting net nominal positions for different age and wealth sectors of the US economy, they look at the redistributive consequences of a switch from a low inflation to a moderate inflation. They find considerable redistributive effects, which happen to be persistent, and larger than any aggregate effects. In their model, redistribution exhibits what they call an “ends-against-the-middle” pattern: the middle class gains at the cost of the rich and poor.

In our model, however, we assume the lack of a large middle class, as can be expected of many developing countries, and, like Albanesi (2002) concentrate on distributional issues between rich and poor.

2. Model I: Regressive Consequences of Asymmetric Information

In this stylized model we isolate and capture the redistributive effect of a monetary shock caused by differing inflationary expectations among agents in the economy, *ceteris paribus*. This model will be the foundation on which the following model is built.

2.1 The Macroeconomic Model

We take a short-run model for a closed and developing economy, on which we will conduct a comparative-static analysis. We denote the first period t_0 , as the given *ex-ante* shock equilibrium state of the economy, E_0 . What we wish to show is that given a disequilibrating change in our model – in the form of a shock to an exogenous variable –, the initial equilibrium will be altered. Consequently, the endogenous variables in our model must undergo an adjustment, which will lead to a new, *ex-post* shock equilibrium state E_t in the second period t . We note that we overlook the process of adjustment of the variables, and simply compare the initial equilibrium state with the final equilibrium state. We also ignore the possibility of instability in the new equilibrium, as we assume that the new (and old) equilibrium is attainable.

In our economy, we have only two representative agents, which we call Rich, \tilde{R} , and Poor, \hat{O} . They are representative of opposite segments of a socioeconomically divided population. The real money supply, M/P , is an increasing function of real output Y , and a decreasing function of the interest rate i . We note here that since this model is purely monetary in nature, we hold output fixed both in the short run, and in the long run, such that $Y = \bar{Y}$. We adopt a Keynesian framework, and will assume ‘sticky’ prices so as to keep the short-run price level, \bar{P} , constant.

We also highlight the long-run price level, P , since some endogenous variables will depend on it. The long-run price level is a function of the short-run money supply so that $P = f(M)$. Wages are also assumed to be sticky in the short-run, so there is no change in our agents’ income within the model. This implies that the change in the money supply does not yet reach our agents’ pockets by the post-shock equilibrium E_t .

Thus, our simple monetary economy, composed of solely of a money market and a fixed goods and services market, is described as follows:

$$\frac{M}{P} = \sigma \bar{Y} - \mu i$$

$$\bar{Y} = \bar{C} + \bar{I} + \bar{G} .$$

We assume that each representative agent can hold only two types of assets: Money, M , i.e. cash or a checking deposit, and a real asset K , e.g. land, a machine, a business or a patent. There is no financial system where agents can allocate their income among interest bearing financial assets, as could be expected in a less-developed country.

Money balances do not yield any return, and provides value insofar as it stores it, and as a medium for transacting. A real asset yields a return insofar as its price rises; it yields no dividends but rather appreciates in nominal value. The price of the real asset, P^K , is assumed to move in proportion with the long run price level, P . Thus, in any long-run equilibrium state, $vP^K = P$. The quantity of real assets, \bar{k} , is fixed because output \bar{Y} is fixed.

2.2 Agent Endowments, Characteristics & Behavior

As mentioned before, we adopt a Keynesian framework, and assume the price level and wages to be sticky during the short-run. However, in the long-run the price level varies, according to the Quantity Theory of Money (QTM):

$$M \bar{V} = P \bar{Y}$$

, where:

V is the velocity of money in final expenditures; assumed fixed.

Thus, the long-run price level changes proportionately to any shock in the money supply induced in the short-run.

We take representative agent \tilde{R} to be a well educated, well informed individual who knows that that the long-run price level will move according to the QTM rule. And since the real asset rises in price along with any long-run changes in the price level P , agent \tilde{R} knows that it will go up in nominal value eventually (thus keeping its real value constant).

We therefore define agent \tilde{R} 's *expected* long-run price level to be a function of the current money supply:

$$\tilde{P}^e = \lambda M$$

, where $\lambda = \frac{1}{\sigma\bar{Y} - \mu i}$.

We note that if the money supply does not change in the short-run, then the short-run price level is in equilibrium with the money supply. In such a case, $\tilde{P}^e = \lambda M = \bar{P}$.

Meanwhile, agent \hat{O} , who we take to be uneducated (e.g. a peasant farmer), and initially ignorant and myopic to any macroeconomic shocks, does not expect the long-run price level to be any different than it is today (i.e. he *expects* a constant price level). His expected long-run price level—irrespective of any changes to the money supply—is a function of the current, and not the expected price level:

$$\hat{P}^e = \bar{P}$$

In sum, in this simple model, the *only* difference between the two representative agents is their expectation of the long-run price level, and thus, the long-run price of the real asset. And we justify this assumed difference on the grounds of informational educational and asymmetries that can be found ubiquitously present between social classes in any developing nation.

2.3 The Real Asset Market

Given inflationary expectations, we now build *ad hoc* demand functions for the real asset for both agents. The demand function for the real asset depends on K 's current price, P^K , and, its expected price as some proportion of the long-run price level. Real asset demand does *not* depend on the interest rate, since money cannot be allocated in interest bearing assets in this model. It does not depend on any wealth stock, or income since these are assumed to be fixed in our model.

Hence, for agent \tilde{R} , the real asset demand function, in real terms, is expressed as follows:

$$\tilde{D}^K = \frac{\alpha + \beta(\tilde{P}^e - P^K)}{\bar{P}}$$

, where:

\tilde{P}^K is the expected long-run price of K , which is proportional to the expected long-run price level

P^K is the current nominal price of the real asset K ;

\bar{P} is the general short-run price level in the economy, which is fixed;

α is a constant;

β is a constant denoting an agent's desire to take advantage of an expected differential between the current price level and the expected future price level. We will assume $\beta > 0$;

The real asset supply, in real terms, is represented by:

$$\tilde{S}^K = \theta \bar{k}$$

, where:

\bar{k} is the fixed quantity of real assets available in the economy;

θ is a constant ($0 \leq \theta \leq 1$) denoting the percentage share of real assets held by agent \tilde{R} .

We set similar equations up for agent \hat{O} , and assume that agent \hat{O} and agent \tilde{R} share the exact same preferences and initial endowments, and only differ in terms of their long-run expectations.

This implies that their coefficients should be identical. Note that from hereon, we normalize the short-run price level \bar{P} to 1. Therefore, the real asset demand function, in real terms, for agent \hat{O} is:

$$\hat{D}^K = \alpha + \beta(\hat{P}^e - P^K)$$

Recall that \hat{P}^e is constant, such that $\hat{P}^e = \bar{P}$.

Similarly, the real asset supply, in real terms, is represented by:

$$\hat{S}^K = (1 - \theta)\bar{k}$$

We now aggregate and equate both agents' real asset demand and supply functions and solve for the equilibrium market price of the real asset, ${}^*P^K$:

First, we sum the both agents' demand functions to get an aggregate demand function:

$$\begin{aligned}\tilde{D}^K + \hat{D}^K &= [\alpha + \beta(\tilde{P}^e - P^K)] + \alpha + \beta(\hat{P}^e - P^K) \\ &= 2\alpha + \beta\tilde{P}^e + \beta\hat{P}^e - 2\beta P^K.\end{aligned}$$

Next we add up both agents' real supply functions for the corresponding aggregate supply function:

$$\tilde{S}^K + \hat{S}^K = \theta\bar{k} + [(1 - \theta)\bar{k}] = \bar{k}$$

We set both equations equal to find the quantity equilibrium for the real asset market:

$$2\alpha + \beta\tilde{P}^e + \beta\hat{P}^e - 2\beta P^K = \bar{k}$$

And now we solve for ${}^*P^K$:

$$\begin{aligned}2\alpha + \beta\tilde{P}^e + \beta\hat{P}^e - 2\beta P^K &= \bar{k} \\ 2\alpha + \beta\tilde{P}^e + \beta\hat{P}^e - 2\beta P^K - \bar{k} &= 2\beta P^K \\ P^K &= \frac{2\alpha}{2\beta} + \frac{\beta\tilde{P}^e}{2\beta} + \frac{\beta\hat{P}^e}{2\beta} - \frac{\bar{k}}{2\beta} \\ {}^*P^K &= \frac{\alpha}{\beta} + \frac{\tilde{P}^e}{2} + \frac{\hat{P}^e}{2} - \frac{\bar{k}}{2\beta}\end{aligned}$$

Recalling that the only variable in the above equation is \tilde{P}^e , and differentiating accordingly, we can see the relationship between the current price of real assets and the expected price by our rich agent:

$$\frac{\partial {}^*P^K}{\partial \tilde{P}^e} = \frac{1}{2}.$$

It follows that if the money supply doubles, the price of capital increases by one half.

We now want to find the equilibrium demand of both agents given the equilibrium price ${}^*P^K$. For this we substitute ${}^*P^K$ into the corresponding equations:

$${}^*\tilde{D}^K = \alpha + \beta(\tilde{P}^e - {}^*P^K)$$

$${}^*\hat{D}^K = \alpha + \beta(\tilde{P}^e - {}^*P^K)$$

Actually substituting in for ${}^*P^K$:

$${}^*\tilde{D}^K = \alpha + \beta \left[\tilde{P}^e - \left(\frac{\alpha}{\beta} + \frac{\tilde{P}^e}{2} + \frac{\hat{P}^e}{2} - \frac{\bar{k}}{2\beta} \right) \right]$$

$$= \alpha + \beta\tilde{P}^K - \alpha - \frac{\beta\tilde{P}^K}{2} - \frac{\beta\hat{P}^e}{2} + \frac{\beta\bar{k}}{2\beta}$$

$$= \frac{\beta\tilde{P}^e}{2} - \frac{\beta\hat{P}^e}{2} + \frac{\bar{k}}{2}$$

$$= \frac{\beta\tilde{P}^e}{2} - \frac{\beta\hat{P}^e}{2} + \frac{\bar{k}}{2}.$$

We repeat the same process for agent \hat{O} and similarly obtain:

$${}^* \hat{D}^K = \frac{\beta \hat{P}^e}{2} - \frac{\beta \tilde{P}^e}{2} + \frac{\bar{k}}{2}$$

We have now obtained the equilibrium demand functions for both agents, which implicitly state that each agent receives the quantity which he demands.

We can now substitute each agent's long-run price expectations into the model to begin to draw conclusions. We recall that \tilde{R} 's expectations are a function of the existing money supply, while \tilde{O} 's expectations are a function of the sticky short-run price level:

$$\tilde{P}^e = \lambda M$$

$$\hat{P}^e = \bar{P}$$

By substitution:

$${}^* \tilde{D}^K = \frac{\beta \lambda M}{2} - \frac{\beta}{2} + \frac{\bar{k}}{2}$$

$${}^* \hat{D}^K = \frac{\beta}{2} - \frac{\beta \lambda M}{2} + \frac{\bar{k}}{2}.$$

By differentiating the equations above, we can draw conclusions on the effect of an exogenous shock to the money supply for both agents. Given a change in M , the change in agent \tilde{R} 's quantity demand for the real assets is:

$$\frac{\partial \tilde{D}^{K*}}{\partial M} = \frac{\lambda \beta}{2}$$

Conversely, for agent \hat{O} we obtain:

$$\frac{\partial \hat{D}^{K*}}{\partial M} = -\frac{\lambda\beta}{2}$$

Given that all parameters are positive, it follows that a positive monetary shock results in:

$$\frac{\partial \tilde{D}^{K*}}{\partial M} > \frac{\partial \hat{D}^{K*}}{\partial M}$$

The above inequality shows that given a positive shock to the money supply, coupled with differing expectations, agent \tilde{R} demands more real assets, while agent \hat{O} demands less. Essentially, \tilde{R} expects a higher price in the long run, and drives up the short-run price of capital by one half the increase in the money supply. This is reflected by:

$$\frac{\partial^* P^K}{\partial \tilde{P}^e} = \frac{1}{2}$$

Meanwhile, agent \hat{O} sees the price of the real asset rise by this amount $(1/2)$, but his expectations have not yet changed. He thinks the long-run price level will be the same as it is today, and thus, views the asset as currently overvalued by \tilde{R} . Not wishing to see a negative return on the asset when it reverts back to its previous value, agent \hat{O} sells his capital to \tilde{R} in what he thinks is a bargain.

Such transactions clearly rearrange each agent's real asset portfolio. To better see each agent's holdings of real assets, W^K , at E_0 and E_t we have:

$$\begin{aligned}\hat{W}_0^K &= \theta \bar{k} \\ \hat{W}_0^K &= (1-\theta)\bar{k} \\ \tilde{W}_t^K &= \theta \bar{k} + \frac{\beta\lambda}{2} \\ \hat{W}_t^K &= \theta \bar{k} - \frac{\beta\lambda}{2}\end{aligned}$$

We will now show that the inverse will hold true for money balances.

2.4 The Money Market

In our model economy, agents can store wealth in either the real assets previously described, or in cash balances. The demand for money depends positively on the current price of assets, as agents need more money to make the same purchases if the current price goes up. And money demand depends negatively on the *expected* change in the long-run price level, as inflation wary agents wish to exit cash balances to avoid any eventual erosion of their purchasing power. The demand for money in this case does not depend on the interest rate, since as previously stated, our underdeveloped economy does not have interest bearing financial assets to serve as alternative investments. And it does not depend on income, since we assume sticky wages in the short-run.

We thus construct the following real money demand function for agent \tilde{R} :

$$\tilde{D}^M = \phi - \eta(\tilde{P}^e - P^K)$$

ϕ is a constant;

η is a constant that shows an agent's desire to exit money balances given expected inflation and the current price of the alternative real asset. We assume $\eta < 0$;

Similarly, the real money supply is represented by:

$$\tilde{S}^M = \tau M .$$

Where:

M is the nominal money supply as controlled by the central bank;

τ is a constant ($0 \leq \tau \leq 1$) representing the percentage share of the nominal money supply held by agent \tilde{R} .

Along the same lines, we obtain the real money demand and supply functions for agent \tilde{O} :

$$\hat{D}^M = \phi - \eta(\hat{P}^e - P^K)$$

$$\hat{S}^M = (1 - \tau)M$$

Since we already have already derived ${}^*P^K = \frac{\alpha}{\beta} + \frac{\lambda M}{2} + \frac{1}{2} - \frac{\bar{k}}{2\beta}$, we can substitute to obtain the equilibrium demand functions for real money balances.

Substituting $\tilde{P}^e = \lambda M$ and $\hat{P}^e = \bar{P}$, for agent \tilde{R} :

$$\begin{aligned}\tilde{D}^M &= \phi - \eta \left(\lambda M - \left(\frac{\alpha}{\beta} + \frac{\lambda M}{2} + \frac{1}{2} - \frac{\bar{k}}{2\beta} \right) \right) \\ {}^*\tilde{D}^M &= \phi - \frac{\eta \lambda M}{2} + \frac{\eta \alpha}{\beta} + \frac{\eta}{2} - \frac{\eta \bar{k}}{2\beta}.\end{aligned}$$

Differentiating to with respect to a change in the money supply:

$$\frac{\partial \tilde{D}^{M*}}{\partial M} = \frac{-\lambda \eta}{2}$$

Similarly for \hat{O} :

$$\begin{aligned}\hat{D}^M &= \phi - \eta \left(\bar{P} - \left(\frac{\alpha}{\beta} + \frac{\lambda M}{2} + \frac{1}{2} - \frac{\bar{k}}{2\beta} \right) \right) \\ {}^*\hat{D}^M &= \phi + \frac{\eta \lambda M}{2} + \frac{\eta \alpha}{\beta} - \frac{\eta}{2} - \frac{\eta \bar{k}}{2\beta}.\end{aligned}$$

Differentiating:

$$\frac{\partial \hat{D}^{M*}}{\partial M} = \frac{\lambda \eta}{2}.$$

We can thus see that, as the money supply increases, agent \hat{O} demands more real money balances than agent \tilde{R} , since:

$$\frac{\partial \hat{D}^{M*}}{\partial M} > \frac{\partial \tilde{D}^{M*}}{\partial M}.$$

This is exactly the opposite of what happens with real assets, as shown in 2.3. Now we calculate the actual change in money stocks, W^M , held by each agent during each period:

$$\begin{aligned}\tilde{W}_0^M &= \tau M \\ \hat{W}_0^M &= (1 - \tau)M \\ \tilde{W}_t^M &= \tau M - \frac{\lambda \eta}{2} \\ \hat{W}_t^M &= (1 - \tau)M + \frac{\lambda \eta}{2}.\end{aligned}$$

We therefore show that the money flows are converse to the real asset flows exhibited in section 2.3, where now agent \tilde{R} has a net outflow of money, which is received by agent \tilde{O} .

2.5 Redistributive Effects in the Long-Run

What we most care about in this paper are the long-run redistributive consequences of an unanticipated monetary shock. We saw that in the new short-run equilibrium E_t , both agents end up holding the same amount of wealth; they just do so in different forms. In this equilibrium, agent \tilde{R} ends up holding more real assets, and agent \hat{O} holds more cash.

However, in the long-run, the price level is due to rise proportionately with the monetary shock induced in the short-run. As we know, if we assume a shock to the money supply, in the short-run $\bar{P} < \lambda M$. But in the long in the long-run, we must necessarily have $P = \lambda M$, where $\bar{P} < P$. This is because P rises to match the rise in M in order to satisfy the long-run equation of the QTM.

We restate a now more relevant point: that the price of the real asset, P^K , rises with the general price level. Thus, in the long-run $\nu P^K = P$.

Now we recall that in short-run trading, agent \tilde{R} purchased more real assets:

$$\frac{\partial \tilde{D}^{K*}}{\partial M} = \frac{\lambda \beta}{2}$$

, while agent \hat{O} received more cash in return:

$$\frac{\partial \hat{D}^{M*}}{\partial M} = \frac{\lambda \eta}{2}$$

The redistribution of wealth takes place as the system enters a long-run equilibrium, where the price level rises. As the price-level breaks its short-run equilibrium and begins to rise, so does the nominal price of real capital. Meanwhile money cannot rise nominally. In strictly nominal terms, the value of the real asset rises:

$$\nu P^K \left(\frac{\beta \lambda}{2} \right)$$

In real terms, and recalling that $\nu P^K = P$, the assets agent \tilde{R} buys from \tilde{O} remain at a constant value:

$$\frac{\nu P^K}{P} \left(\frac{\beta \lambda}{2} \right) = \frac{P}{P} \left(\frac{\beta \lambda}{2} \right) = \left(\frac{\beta \lambda}{2} \right)$$

On the other hand, the cash agent \hat{O} receives does not appreciate nominally like a real asset, and it must lose value in real terms:

$$\frac{1}{P} \left(\frac{\eta \lambda}{2} \right)$$

Thus, we can see that the assets \tilde{R} acquired from \hat{O} keep their real value constant by rising in price in parallel with an increase in the general price level. In nominal terms, K increases in price. However, the money that \hat{O} acquired from \tilde{R} stays at a constant nominal value, and in fact loses real value as inflation erodes its purchasing power. It is thus shown that in relative terms, \tilde{R} has thus gained wealth, at the expense of agent \hat{O} 's naïveté.

2.7 A Numerical Example Using Balance Sheets

Below we show the effect in this section's model using a balance sheet approach.

		Rich			Poor			
		Assets	Liabilities	Net	Assets	Liabilities	Net	
t = 0	Initial Endowments	50 K 50 M		100	50 K 50 M		100	Total 200
	Shock to M (2M)							
t = 1	Money flows to Poor	- 50 M		50	+ 50 M		150	
	Real Asset flows to Rich	+50 K		100	- 50 K		100	
t = 2	M increase is received via wage rise	+ 50 M		150	+50 M		150	
	Price Level Rises (2P)	50 / 2 M (2 x 100) / 2 K		25 M 100 K	150 M / 2 0 K		75 M 0 M	Total 200
		Real Wealth = 125 (+25)			Real Wealth 75 (-25)			

At time t_0 , both agent have endowments of cash M and real assets K . We can assume for simplicity of calculation that both agents have the same initial wealth portfolio evenly divided among 50 units of real assets and 50 units of money. The money supply is thus 100, accounting for both

agents, and the total amount of real wealth in the economy is 200. At this time, the short-run price level and the money supply are in equilibrium, and therefore both agents have the same expectations, and the economy is at rest.

If we double the money supply between t_0 and time t , we see how agent \tilde{R} uses his money stock to buy up real capital, while conversely, \tilde{O} accumulates cash, and sells his real assets. In this simple example, \tilde{R} unloads all his money holdings, and buys 50 units worth of real assets. \tilde{O} accepts 50 units of real money in return.

We can see that since the price level is still fixed in the short-run, both agents still hold the same amount of real wealth (i.e. 100 each). Up to this point, the only evident change lies in the composition of both agents' wealth portfolio (a swap of assets).

However, in the long run, prices rise in proportion to the initial increase in money. We can assume that agents start equally receiving the government's money increase through wages, which in turn helps push prices up soon thereafter. Each agent thus receives 50 nominal units of the money supply increase.

But the increase in money wealth is only nominal, as the rise in prices quickly erodes its purchasing power. The 50 nominal units in money income received by each agent are divided by a doubling of the price level, so that their real value is only 25. Agent \tilde{O} 's money holdings suffer the same fate, and are eroded by the rise in the price level, leaving him with a real money balance of 25 (from the 50 nominal units) and 50 (from the 100 nominal units carried over from last period). His total real wealth now stands at 75.

Noting that real wealth cannot be created nor destroyed via monetary disturbances, the 25 units of real wealth lost by agent \tilde{O} must necessarily now be held by \tilde{R} . And this is precisely the case: \tilde{R} 's real capital rose along with the price level, thus offsetting any erosion caused by inflation. In other words, his 100 units in real capital holdings doubled in price to 200 nominal units; as the price level doubles from 1 to 2, real wealth stays constant at 100 units. Add his 25 units of real money income from the increase to the money supply, and he now has 125 units of real wealth relative to \tilde{O} 's 75. There has been an effective concentration of wealth in favor of agent \tilde{R} .

3. Model II: Regressive Consequences of Imperfect Capital Markets

In this model, we build on the previous model by adding a new asymmetry between agents, this time in the form of imperfect capital markets. We keep the redistributive effect caused by the confluence of a monetary shock and differing inflationary expectations among agents in the economy, and now add the nominal wealth effect that comes with access to domestic capital markets by only one group of agents. We argue that capital markets provide those fortunate enough to access them increased buying power at the time of a monetary shock. This in turn further exacerbates the regressive distributional effects noted in the first model.

We assume that the economy has a fixed outstanding quantity of financial assets, all of which are held by representative agent \tilde{R} . Agent \tilde{O} is locked out of the financial system, and can still only hold cash balances. This is typical in developing economies where financial depth is often very low, and limited to urban and or more privileged sectors of the population.

3.1 The Macroeconomic Model

This macroeconomic model is identical to the previous one, except for the fact that now we include a fixed amount of interest bearing assets in our economy. We define these instruments, e.g. discount bonds, B , as functional of the nominal interest rate:

$$B = \frac{\varpi}{1+i}$$

Where:

- B is the nominal value of bond holdings;
- ϖ is a constant representing the face value;
- i is the nominal interest rate.

We note here that i is endogenous. Keeping in mind that $i = \frac{\sigma \bar{Y}}{\mu} - \frac{M}{\mu \bar{P}}$, an increase in the money supply will cause interest rates to fall in the short-run, which in turn will spur the appreciation of bonds.

3.2 Agent Endowments, Characteristics & Behavior

We keep the informational asymmetry that leads to lagging expectations on behalf of agent \tilde{O} , and place a new restriction upon him: he cannot access interest-bearing financial instruments. Therefore, the newly introduced difference between agents is reflected in their wealth holdings. Agent \tilde{O} has a fixed wealth portfolio of cash, \hat{W} . It is invariant to any movements in the interest rate. However, agent \tilde{R} is considerably more fortunate, and can invest his cash into financial assets. This implies that \tilde{R} 's wealth portfolio, held in bonds, is a function of the nominal interest rate that prevails in the economy. Therefore $\tilde{W} = \tilde{W}[B(i)]$. This implies that if there is a shock to the money supply, the interest rate will fall, and the wealth holdings of \tilde{R} jump up in nominal value in the short-run. Upon comparison, we can see that this is the only modification we make to our base model, but it will prove to be a significant one.

3.3 The Real Asset Market

We again keep the same characteristics of the real asset demand function from Section 2, but now we add a wealth component for agent \tilde{R} which is variable as a function of the interest rate so that $\tilde{W} = \tilde{W}[B(i)]$, as noted above. As we highlighted, representative agent \tilde{R} holds *all* bonds in the economy, as agent \hat{O} lacks the means and knowledge to access them. This implies that the demand function below does not depend negatively on the interest rate, since agent \tilde{R} already holds all the bonds he can hold; he will not, and cannot hold more bonds even if the interest rate rises. This also implies that we will not build a financial asset market, given that demand is necessarily fixed, and restricted to one of the groups.

Normalizing the short-run price level to $\bar{P} = 1$, for agent \tilde{R} , the real asset demand, in real terms, looks as follows:

$$\tilde{D}^K = \alpha + \beta(\tilde{P}^e - P^K) + \delta\tilde{W}$$

, where:

\tilde{W} is the agent's wealth as held in bonds;

The real asset supply, in real terms, is still represented by:

$$\tilde{S}^K = \theta\bar{k}.$$

The real asset demand function, in real terms, for agent \hat{O} is:

$$\hat{D}^K = \alpha + \beta(\hat{P}^e - P^K) + \delta\hat{W}.$$

Similarly, the real asset supply, in real terms, is represented by:

$$\hat{S}^K = (1 - \theta)\bar{k}.$$

Next we derive the equilibrium market price of the real asset, P^{K^*} to be:

$$*P^K = \frac{\alpha}{\beta} + \frac{\tilde{P}^e}{2} + \frac{\hat{P}^e}{2} + \frac{\delta\tilde{W}}{2\beta} + \frac{\delta\hat{W}}{2\beta} - \frac{\bar{k}}{2\beta}.$$

We obtain the equilibrium demand both agents:

$$\tilde{D}^K = \alpha + \beta \left(\tilde{P}^e - \left(\frac{\alpha}{\beta} + \frac{\tilde{P}^e}{2} + \frac{\hat{P}^e}{2} + \frac{\delta\tilde{W}}{2\beta} + \frac{\delta\hat{W}}{2\beta} - \frac{\bar{k}}{2\beta} \right) \right) + \delta\tilde{W}$$

$$* \tilde{D}^K = \frac{\beta \tilde{P}^e}{2} - \frac{\beta \hat{P}^e}{2} + \frac{\delta \tilde{W}}{2} - \frac{\delta \hat{W}}{2} + \frac{\bar{k}}{2}.$$

Now we derive the rate of change in agent \tilde{R} 's quantity demand \tilde{D}^{K*} for real assets, given a shock to M . First, we make the above equilibrium demand a function of M . For this we define the value of wealth stock in terms of M by plugging in $i(M)$ into $B(i)$:

$$B = \frac{\varpi}{1+i}$$

$$i = \frac{\sigma \bar{Y}}{\mu} - \frac{M}{\mu \bar{P}}$$

$$\tilde{W}(M) = \frac{\varpi}{1 + \left(\frac{\sigma \bar{Y}}{\mu} - \frac{M}{\mu \bar{P}} \right)}.$$

With this we plug in for \tilde{P}^e and $\tilde{W}(M)$:

$$* \tilde{D}^K = \frac{\beta \lambda M}{2} - \frac{\beta}{2} + \frac{\delta \left(\frac{\varpi}{1 + \left(\frac{\sigma \bar{Y}}{\mu} - \frac{M}{\mu \bar{P}} \right)} \right)}{2} - \frac{\delta \hat{W}}{2} + \frac{\bar{k}}{2}.$$

We now have an equilibrium demand function in terms of the only exogenous variable M .

To arrive at $\frac{\partial \tilde{W}^* \tilde{D}^K}{\partial M}$ as we did in section 2, we first isolate and differentiate the more complicated

term $\frac{\partial \tilde{W}(M)}{\partial M}$:

$$\begin{aligned}
 \tilde{W}(M) &= \delta \left(\frac{\varpi}{1 + \left(\frac{\sigma\bar{Y}}{\mu} - \frac{M}{\mu\bar{P}} \right)} \right) \\
 &= \delta \left(\frac{\varpi}{1 + \left(\frac{\sigma\bar{Y}}{\mu} - \frac{M}{\mu} \right)} \right) \\
 &= \frac{\delta\varpi}{\left(1 + \frac{\sigma\bar{Y}}{\mu} - \frac{M}{\mu} \right)}
 \end{aligned}$$

Differentiating:

$$\begin{aligned}
 \frac{\partial \tilde{W}(M)}{\partial M} &= \frac{(\delta\varpi)' \left(1 + \frac{\sigma\bar{Y}}{\mu} - \frac{M}{\mu} \right) - (\delta\varpi) \left(1 + \frac{\sigma\bar{Y}}{\mu} - \frac{M}{\mu} \right)'}{\left(1 + \frac{\sigma\bar{Y}}{\mu} - \frac{M}{\mu} \right)^2} \\
 &= \frac{\left(\frac{\delta\varpi}{\mu} \right)}{\left(1 + \frac{\sigma\bar{Y}}{\mu} - \frac{M}{\mu} \right)^2}.
 \end{aligned}$$

Therefore:

$$\frac{\partial \tilde{D}^{K^*}}{\partial M} = \frac{\beta}{2} + \frac{\left(\frac{\delta\varpi}{\mu} \right)}{\left(1 + \frac{\sigma\bar{Y}}{\mu} - \frac{M}{\mu} \right)^2}.$$

We do the same for agent \hat{O} :

$${}^* \hat{D}^K = \frac{\beta}{2} - \frac{\beta \lambda M}{2} - \frac{\delta \left(\frac{\varpi}{1 + \left(\frac{\sigma \bar{Y}}{\mu} - \frac{M}{\mu \bar{P}} \right)} \right)}{2} + \frac{\delta \hat{W}}{2} + \frac{\bar{k}}{2}$$

$$\frac{\partial \hat{D}^{K*}}{\partial M} = -\frac{\beta}{2} - \frac{\left(\frac{\delta \varpi}{\mu} \right)}{\left(1 + \frac{\sigma \bar{Y}}{\mu} - \frac{M}{\mu} \right)^2}.$$

Given that all parameters are positive, it again it follows that::

$$\frac{\partial \tilde{D}^{K*}}{\partial M} > \frac{\partial \hat{D}^{K*}}{\partial M}.$$

Thus, we again show that given a shock to the money supply, agent \tilde{R} demands more real assets than does agent \hat{O} , this time by differing inflationary expectations *and* by the nominal wealth effect produced by appreciating bonds. This means that, relative to our first model, even *more* real assets are transferred to agent \tilde{R} . Agent \hat{O} now gets even more cash from \tilde{R} , as the latter now has more buying power and uses it.

3.4 The Money Market

The money demand functions remain the same as in the first model, however, again we incorporate the variable wealth component for agent \tilde{R} . We thus construct the following real money demand function:

$$\tilde{D}^M = \phi - \eta(\tilde{P}^e - P^K) + \gamma \tilde{W}$$

We keep the previous real money supply:

$$\tilde{S}^M = \tau M$$

Now the real money demand function for agent \tilde{O} :

$$\hat{D}^M = \phi - \eta(\hat{P}^e - P^K) + \gamma \hat{W}$$

and,

$$\hat{S}^M = (1 - \tau)M$$

We substitute M into all corresponding terms, arrive at ${}^*\tilde{D}^M$ and differentiate to get the change given a money supply shock. The equilibrium money demand for agent \hat{O} is:

$$\tilde{D}^M = \phi - \eta \left(\lambda M - \left(\frac{\alpha}{\beta} + \frac{\lambda M}{2} + \frac{1}{2} + \frac{\delta \tilde{W}}{2\beta} + \frac{\delta \hat{W}}{2\beta} - \frac{\bar{k}}{2\beta} \right) \right) + \gamma \tilde{W}$$

Now as a function of M :

$${}^*\tilde{D}^M = \phi - \frac{\eta \lambda M}{2} + \frac{\eta \alpha}{\beta} + \frac{\eta}{2} + \frac{\eta \delta \left(\frac{\varpi}{1 + \left(\frac{\sigma \bar{Y}}{\mu} - \frac{M}{\mu} \right)} \right)}{2\beta} + \frac{\delta \hat{W}}{2\beta} - \frac{\eta \bar{k}}{2\beta} + \gamma \left(\frac{\varpi}{1 + \left(\frac{\sigma \bar{Y}}{\mu} - \frac{M}{\mu} \right)} \right)$$

$$= \phi - \frac{\eta \lambda M}{2} + \frac{\eta \alpha}{\beta} + \frac{\eta}{2} + \frac{\Gamma \left(\frac{\varpi}{1 + \left(\frac{\sigma \bar{Y}}{\mu} - \frac{M}{\mu} \right)} \right)}{2\beta} + \frac{\delta \hat{W}}{2\beta} - \frac{\eta \bar{k}}{2\beta}$$

,where $\Gamma = \eta\delta + \gamma 2\beta$.

Differentiating:

$$\frac{\partial \tilde{D}^{M*}}{\partial M} = \frac{-\eta}{2} + \frac{\left(\frac{\Gamma \varpi}{\mu}\right)}{\left(2\beta + \frac{2\beta\sigma\bar{Y}}{\mu} - \frac{2\beta M}{\mu}\right)^2}.$$

For agent \hat{O} :

$${}^* \hat{D}^M = \phi + \frac{\eta\lambda M}{2} + \frac{\eta\alpha}{\beta\bar{P}} - \frac{\eta}{2} + \frac{\delta \left(\frac{\varpi}{1 + \left(\frac{\sigma\bar{Y}}{\mu} - \frac{M}{\mu}\right)} \right)}{2\beta\bar{P}} + \frac{\delta\hat{W}}{2\beta} - \frac{\eta\bar{k}}{2\beta}.$$

Differentiating:

$$\frac{\partial \hat{D}^{M*}}{\partial M} = \frac{\eta}{2} + \frac{\left(\frac{\delta\varpi}{\mu}\right)}{\left(2\beta + \frac{2\beta\sigma\bar{Y}}{\mu} - \frac{2\beta M}{\mu}\right)^2}.$$

We can thus see that, as the money supply increases, agent \hat{O} demands more real money balances than agent \tilde{R} , since:

$$\frac{\partial \hat{D}^{M*}}{\partial M} > \frac{\partial \tilde{D}^{M*}}{\partial M}.$$

Here Agent \tilde{R} keeps some percentage of his income, inversely proportional to δ , his marginal propensity to purchase real assets given an increase in wealth. The remaining money is sold to \tilde{O} , who sells his real asset in return. As the long run price level again rises, the redistributive consequences parallel those seen in our first model, only that the effects are aggravated by coupling added purchasing power to more accurate expectations on the long-run effects of inflation..

4. Conclusions

In this paper we have shown two channels by which a nation's existing wealth can be distributed to those who have informational and financial advantages. Such asymmetries can exist anywhere, but due to educational gaps and underdeveloped financial systems, we believe that their effects may magnify polarization of real capital in developing nations. Since income is the return to capital, it would therefore imply a redistribution of income in the long-run.

Recurrent monetary shocks and inflationary conditions are thus shown to have a regressive effect that exhibits properties of hysteresis once the proposed system enters a steady state. This coincides with the findings in Doepke and Schnieder (2003). This implies that unless the distributive system is 'counter-shocked', the distribution of wealth will have become permanently more concentrated. And given that such shocks are intermittent so that poor agents don't become accustomed and learn (i.e. quickly adjust expectations), over time, several shocks can spur the recursive accumulation of wealth by informed and financially advantaged agents.

4.1 Policy Implications

This analysis highlights the importance of sound and predictable monetary policy for the central bankers of developing nations. Monetary policy must be congruent with governmental efforts to not only to ensure full and stable employment, but also to assure balanced growth. Though the effects of inflation on the salaries of poor and middle class households are well know by policy makers, long-run redistributive properties of wealth should be kept in mind when increasing the money supply for common purposes such as the monetization of nominal debt. Provided that the effects in this paper can be confirmed, the regressive properties of the money supply increase should later be

offset by some type of compensatory wealth tax. This is the aforementioned ‘counter-shock’ to the system of wealth distribution.

4.2 Extensions

There are two roads via which to extend this paper. The first is to improve the model itself, by developing an optimization framework for the functions we provided *ad hoc*. A continuous inter-temporal general equilibrium model, with a production and government sector would be in order. The imperfect capital markets assumption could be extended to a model of a small open economy with imperfectly substitutable assets, where some agents can access foreign assets, while poorer agents are limited to assets denominated in domestic currency. In such a case, a monetary shock, and thus devaluation, would have relative wealth effects via the exchange rate.

It would also be important to empirically evaluate the hypothesized mechanisms presented herein. It would be important to test whether this relationship exists. Given limited data on distribution of wealth, income distribution would likely have to be used as a proxy, though this would present some complications. We leave this open for further investigation.

References

- Aiyagari, S. Rao. 1994. "Uninsured Idiosyncratic Risk and Aggregate Saving." *Quarterly Journal of Economics* 109 (3): 659–84.
- Albanesi, Stefania. 2002. "Inflation and Inequality." Unpublished Manuscript, Bocconi University.
- Bahmani-Oskoei, Goswami and Solomon Mebratu. 2006. "Black Market Premium and Income Distribution." *The Journal of Developing Areas* 39: 2
- Chiang, Alpha C. 1984. *Fundamental Methods of Mathematical Economics*. New York: McGraw-Hill
- Calvo, Guillermo. 2000. "Notes on Price Stickiness: With Special Reference to Liability Dollarization and Credibility" *University of Maryland & NBER Working Paper*.
- Doepke, Matthias and Martin Schneider 2006. "The Real Effects of Inflation through the Redistribution of Nominal Wealth." Working paper, UCLA & NYU.
- Erosa, Andres and Gustavo Ventura. 2002. "On Inflation as a Regressive Consumption Tax." *Journal of Monetary Economics* 49:761–95.
- Flavin, Marjorie and Takashi Yamashita 2002. "Owner-Occupied Housing and the Composition of the Household Portfolio." *American Economic Review*, pp. 345-62.
- Glaeser, Edwards, Joseph Gyourko, and Raven E. Saks 2005. "Why have House Prices Gone Up?" Working paper, Harvard University and Wharton.
- Knight, J.B. 1976. "Devaluation and Income Distribution in Less-Developed Economies." *Oxford Economic Papers*, New Series, Vol. 28, No.2 (Jul., 1976), pp. 208-227.
- Krugman, Paul R. and Maurice Obstfeld. 2000. *International Economics: Theory and Policy*. New York: Addison-Wesley.
- Mankiw, Gregory N., Ricardo Reis and Justin Wolfers 2003. "Disagreement about Inflation Expectations". NBER Working paper 9796.
- Persson, Mats, Torsten Persson, and Lars E. O. Svensson. 1998. "Debt, Cash Flow and Inflation Incentives: A Swedish Example." In *The Debt Burden and its Consequences for Monetary Policy*, edited by Guillermo Calvo and Mervyn King, 28–62.

New York: St. Martin's Press.

Romer, David. 2006. *Advanced Macroeconomics*. Boston: McGraw-Hill

Silberberg, Eugene. 1990. *The Structure of Economics: a Mathematical Analysis*. New York: McGraw-Hill

Varian, Hal R. 2003. *Intermediate Microeconomic: A Modern Approach*. New York: Norton